CS 6110 Problem Set 3 Hints for Problem 2 in Lecture 26

Show $x * y = y * x$. Recall by definition $\begin{cases} 0 * y = 0 \text{ and} \\ Sx * y = (x * y) + y \end{cases}$

Note that we already proved

1. $(x+y) = (y+x)$ and we can prove

2. $(x+y)+z = x+(y+z)$. You can now assume this.

Also note that $x * Sy$ and $x*y + x$ both satisfy the equation

3. $f(Sx, y) = f(x,y) + Sy$

To say that a function f satisfies an equation is to substitute it into the equation. Here is how to show that $\lambda x,y.(x * Sy)$ satisfies the equation for f in 3.

$$\underbrace{\lambda x,y.(x * Sy)\,(Sx)(y)}_{\parallel} = \lambda x,y.(x * Sy)(x,y) + Sy$$

$$Sx * Sy = x * Sy + Sy \quad \text{that is}$$

$$(x+1) * (y+1) = x * (y+1) + (y+1)$$

$$x*y + x + y + 1 = x*y + x + y + 1$$

4. As announced in class, if there are very basic facts about addition, as 1, 2, above, you may assume them. Be explicit about the assumptions. You can assume facts about multiplication as long as you don't trivialize the problem.